

pSHAKE

MANUAL PROBABILISTIC SITE RESPONSE ANALYSES WITH SHAKE91

PREPARED BY

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0. UPDATES AND DOWNLOAD

■ UPDATES

- Oct-30-2018 Using the “scatter” command for plotting the mean amplification factor (MAF)
Checking the time consuming for each cases
- Oct-10-2018 Change format for COV of V, B and B (8th line - Format: I5, 3I10)
Plot nonlinear soil in case without randomization (5th line, column 1-5)
Update option o for iSASSI (7th line, column 11-15)
- Apr-23-2018 Excel file is generated to save all output data.
- Apr-20-2018 Output file name of nonlinear soil has been changed.
- Oct-30-2017 Layering and velocity model (Toro, 1995) has been added.
- Oct-13-2017 SASSI2010 input files are generated.
- Oct-05-2017 Simulated Profiles (After Low-Strain Realizations) on fig2
- Sept-01-2017 Simulated Profiles (Before Low-Strain Realizations) on fig1

■ DOWNLOAD

- + pShake.zip: <https://www.dropbox.com/s/cpvx7lowlwpxfsd/pshake.zip?dl=0>
- + Execution: Run "pShake.exe"
- + Input File (Sample): (e.g. 'control.inp' with 'shake.dat' and 'diam.acc')
- + Output File (Sample): (e.g. shake.o1, shake.o2, shake.png)

■ We will appreciate if you refer the following material for pShake

1. Tran, T. T., Han, S. R., & Kim, D. (2018). Effect of probabilistic variation in soil properties and profile of site response. *Soils and Foundations*, 58(6), 1339-1349.
2. Tran, T. T., Salman, K., Han, S. R., & Kim, D. (2020). Probabilistic Models for Uncertainty Quantification of Soil Properties on Site Response Analysis. *ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering*, 6(3), 04020030.
3. Tran, T. T., Nguyen, P. C., Han, S. R., & Kim, D. (2020). Stochastic Site Response Analysis in Consideration with Various Probability Distributions of Geotechnical Properties. In *CIGOS 2019, Innovation for Sustainable Infrastructure* (pp. 901-906). Springer, Singapore.

1. INTRODUCTION

+ The program, pShake, was developed by the results of the consultation by KEPCO-ENC in November 2017, "Development of probabilistic site response analysis program for earthquake response analysis (I)".

+ Details of the consultation and the results of the consultation are summarized as follows.

1. Probabilistic Site Response Analysis () Methodology Survey and Related Design Criteria Survey
- Chapter 2 Seismic Input in ASCE 4-16 Seismic Analysis of Safety-Related Nuclear Structures

2. Development of Soil Damping Relationship and Probabilistic Variable Extraction Algorithm for Probabilistic Site Response Analysis

- In order to realize probabilistic low strain rate, the method of estimating the random variables related to the shear modulus (G) and the damping ratio (D) is summarized in the G/Gmax curve and the attenuation curve data.

- In addition, SHAKE91, a representative site response analysis program, was analyzed and improved to select random variables for site response analysis.

3. Probabilistic site response analysis: Input data, analysis, output data

- PShake, an automation program for input data, analysis, and output data for stochastic site response analysis, and the theories, manuals, and examples of this program are included in this report.

2. THEORY: VARIATION OF SITE PROPERTIES

2.1 Introduction

+ A soil profile consists of discrete layers that vary in thickness based on the properties of the soil. The layers are typically discretized based on the soil type, recorded from borehole samples or inferred from a shear wave velocity profile.

In seismic site response analysis, each layer is characterized by a thickness, mass density, shear wave velocity, and nonlinear properties (G/G_{max} , and D).

One of the challenges in defining values for these properties is the natural variability across a site and the uncertainty in their measurement.

Because the dynamic response of a site is dependent on the soil properties, any variation in the soil properties will change both the expected surface motion and its standard deviation.

+ In a simple system, the variability of the components can be analytically combined to quantify the variability of the complete system, thus allowing for the expected value and variability of the system response to be computed.

In seismic site response analysis, the nonlinear response of the system does not allow an exact analytic quantification of the variability of the site response.

Instead, an estimate of the expected surface response and its standard deviation due to variations in the soil properties can be made through Monte Carlo simulations.

Monte Carlo simulations estimate the response of a system by generating parameters of the system based on defined statistical distributions and computing the response for each set of input parameters.

The following chapter introduces Monte Carlo simulations as applied to site response analysis and presents the models that describe the variability of the layering, shear wave velocity, and nonlinear properties (G/G_{max} , and D).

2.2 Random Variables

+ The goal of a Monte Carlo simulation is to estimate the statistical properties of the response of a complex system.

To achieve this goal, each of the properties of the system is selected from defined statistical distributions and the response of the system is computed.

The response is computed for many realizations and the calculated response from each realization is then used to estimate statistical properties of the system's response.

While Monte Carlo simulations can be used on a wide variety of problems, a major disadvantage is that a large number of simulations is required to achieve stable results.

- + Monte Carlo simulations require that each of the components in the system has a complete statistical description.

The description can be in the form of a variety of statistical distributions (i.e., uniform, triangular, normal, log-normal, exponential, etc.);

however the normal and log-normal distributions typically are used because they can be easily described using a mean (μ) and a standard deviation (σ).

For normally distributed variables, a random value (x) can be generated by:

$$x = \mu_x + \sigma_x \varepsilon \quad (2.2.1)$$

where μ_x is the mean value, σ_x is the standard deviation, and ε is a random variable with zero mean and unit standard deviation.

Random values of ε are generated and used to define the random values of x .

- + To generate multiple random variables that are independent, Eq. (2.2.1) can be used for each variable with different, random values of ε generated for each variable.

In the case of correlated random variables, a more complicated procedure is required for the generation of values.

The correlation between variables is quantified through the correlation coefficient (ρ).

The correlation coefficient can range from -1 to 1.

Uncorrelated variables have $\rho = 0$ (Fig. 2.2.1a).

Positive correlation between variables indicates that the two variables have a greater tendency to both differ from their respective mean values in the same direction (Fig. 2.2.1b).

As ρ approaches 1.0, this correlation becomes stronger.

Negative correlation indicates that variables have a greater tendency to differ in the opposite direction (Fig. 2.1c).

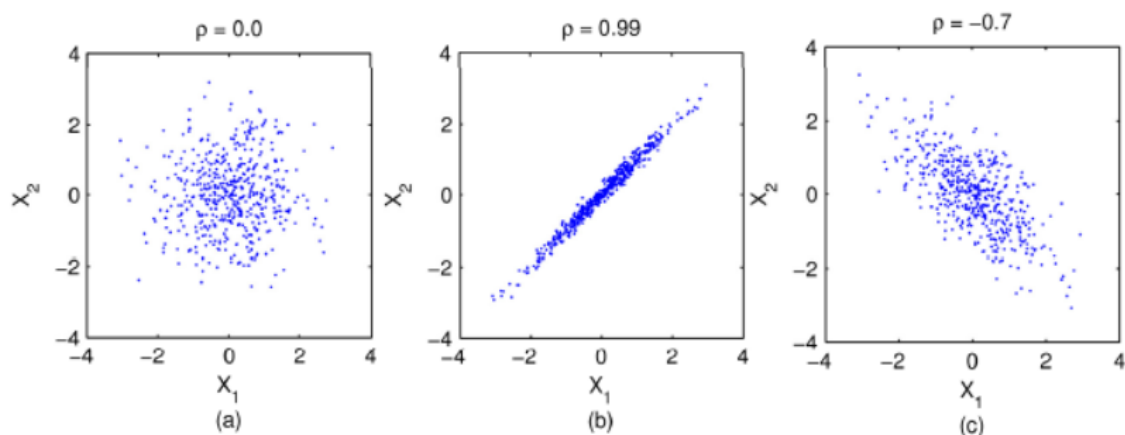


Fig. 2.2.1 Two variables with different correlation coefficients

+ As discussed previously, independent random variables from a normal distribution are generated by applying Eq. (2.2.1) independently to each random variable.

By combining the multiple applications of Eq. (2.2.1) into a system of equations, the generation of two independent variables is achieved by multiplying a vector of random variables ($\vec{\varepsilon}$) by a matrix ($[\sigma]$) and adding a constant ($\vec{\mu}$), defined as:

$$\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{bmatrix} \sigma_{x_1} & 0 \\ 0 & \sigma_{x_2} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{Bmatrix} + \begin{Bmatrix} \mu_1 \\ \mu_2 \end{Bmatrix} \quad (2.2.2)$$

where ε_1 and ε_2 are random variables randomly selected from a standard normal distribution ($\mu = 0$ and $\sigma = 1$), σ_{x_1} and σ_{x_2} and are the standard deviations of x_1 and x_2 , respectively, and μ_1 and μ_2 are the mean values of x_1 and x_2 , respectively.

Because the random variables x_1 and x_2 are independent ($\rho_{x_1, x_2} = 0$), the off-diagonal values in the matrix ($[\sigma]$) are zero.

+ Using the same framework, a linear system of equations is used to generate a pair of correlated random variables.

However, the off-diagonal values in the matrix can no longer be zero because of the correlation between x_1 and x_2 .

Instead, a pair of correlated random variables (\vec{x}) is generated by (Kao 1997):

$$\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{bmatrix} \sigma_{x_1} & 0 \\ \rho_{x_1 x_2} \sigma_{x_2} & \sigma_{x_2} \sqrt{1 - \rho_{x_1 x_2}^2} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{Bmatrix} + \begin{Bmatrix} \mu_1 \\ \mu_2 \end{Bmatrix} \quad (2.2.3)$$

Here, the first random variable (x_1) is calculated based on the value of ε_1 alone, while the second random variable (x_2) is a function of both ε_1 and ε_2 .

Note that ε_1 and ε_2 still represent random and independent variables generated from the standard normal distribution.

2.3 Layering and Velocity Model (Toro, 1995)

+ For the properties of the soil to be randomized and incorporated into Monte Carlo simulations, the statistical distribution and properties of the soil need to be characterized.

In this research, two separate models are used.

The first model, developed by Toro (1995), describes the statistical distribution and correlation between layering and shear wave velocity.

The second model by Darendeli (2001) is used to describe the statistical distribution of the nonlinear properties (G/G_{max} , and D).

+ In pShake, the randomizations of the layering and the shear wave velocity are done through the use of the models proposed by Toro (1995).

The Toro (1995) models provide a framework for generating layering and then to vary the shear wave velocity of these layers.

The model for shear wave velocity variation improves upon previous work by quantifying the correlation between the velocities in adjacent layers.

In previous models, one of two assumptions were made that simplified the problem:

the velocities at all depths are perfectly correlated and can be randomized by applying a constant random factor to all velocities (McGuire et al. 1989; Toro et al. 1992), or

the velocities within each of the layers are independent of each other, and therefore can be randomized by applying an independent random factor to each layer (Costantino et al. 1991).

While these two assumptions simplify the problem, they represent two extreme conditions.

The Toro (1995) model makes neither of these assumptions; instead the model incorporates correlation between layers.

2.3.1 Homogeneous Layering

+ The layering is modeled as a Poisson process*, which is a stochastic process with events occurring at a given rate (λ).

For a homogeneous Poisson process this rate is constant, while for a nonhomogeneous Poisson process the rate varies.

Generally, a Poisson process models the occurrence of events over time, but for the layering problem the event is a layer interface and its rate is defined in terms of length (i.e., number of layer interfaces per meter)†.

+ In the Toro (1995) model, the layering thickness is modeled as a nonhomogeneous Poisson process where the rate changes with depth ($\lambda(d)$, where d is depth from the ground surface).

Before considering the nonhomogeneous Poisson process, first consider the simpler homogeneous

* 포아송분포(Poisson distribution)

- 어떤 주어진 시간내(또는 공간내)의 발생빈도 λ 로 어떤 독립적인 베르누이 사건이 일어날 기댓값을 u 라고 할 때, u 는 양의 정수로 다음과 같은 확률밀도함수(PDF)와 누적분포함수(CDF)로 나타낼 수 있다.

$$f(u) = \frac{\lambda^u e^{-\lambda}}{u!}, \quad F(u) = \sum_{i=0}^u \frac{\lambda^i e^{-\lambda}}{i!}$$

- 포아송분포의 평균과 분산은 동일한 값으로 다음과 같다.

$$\mu_u = \sigma_u^2 = \lambda$$

- 포아송분포는 기본적으로 서로 독립인 베르누이 사건이 시간 구간 등 어떤 구간에서 연속하여 발생하는 경우에 대한 분포이므로, 지진이나 폭풍우의 발생, 교차로에 도착하는 자동차의 수와 같이 연속하여 발생하는 사건의 결과가 그 이전에 발생한 사건들의 결과와 전혀 무관할 경우에(즉, 독립일 경우에), 분포로 이용할 수 있다.

† 발생빈도 $\lambda = 1$, 기대 층두께 (또는 재현 층두께) $h = 1/\lambda = 1$ m.

발생빈도 $\lambda = 0.5$, 기대 층두께 (또는 재현 층두께) $h = 1/\lambda = 2$ m.

발생빈도 $\lambda = 0.2$, 기대 층두께 (또는 재현 층두께) $h = 1/\lambda = 5$ m.

Poisson process with a constant rate.

For a Poisson process with a constant occurrence rate (λ), the distance between layer boundaries, also called the layer thickness (h), has an exponential distribution with rate λ .

The probability density function (PDF) of an exponential distribution is defined as (Ang and Tang 1975):

$$f(h; \lambda) = \begin{cases} \lambda e^{-\lambda h} & , h \geq 0 \\ 0 & , h < 0 \end{cases} \quad (2.3.1)$$

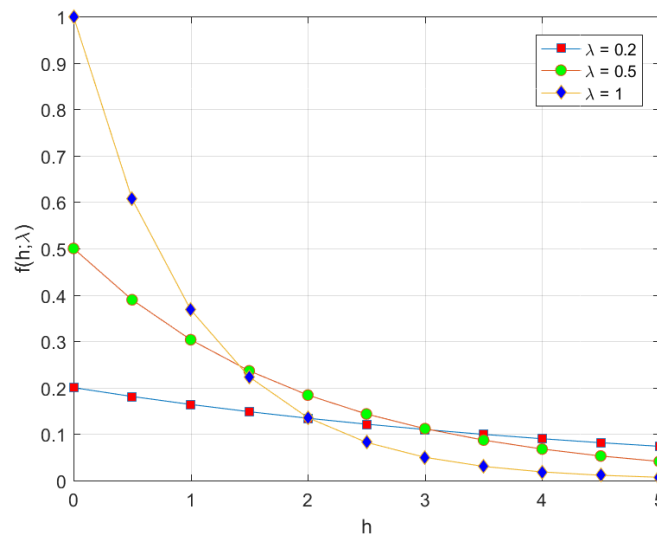


Figure 4.3.1 PDF of Poisson's distribution

+ The cumulative density function (CDF) for the exponential distribution is given by:

$$F(h; \lambda) = \begin{cases} 1 - e^{-\lambda h} & , h \geq 0 \\ 0 & , h < 0 \end{cases} \quad (2.3.2)$$

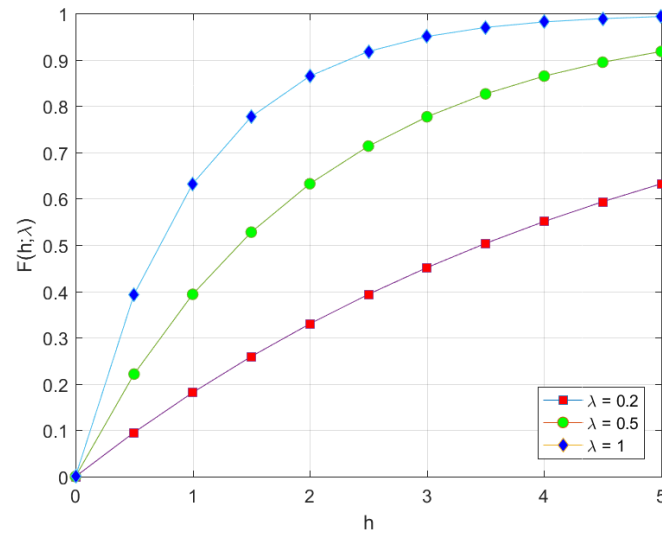


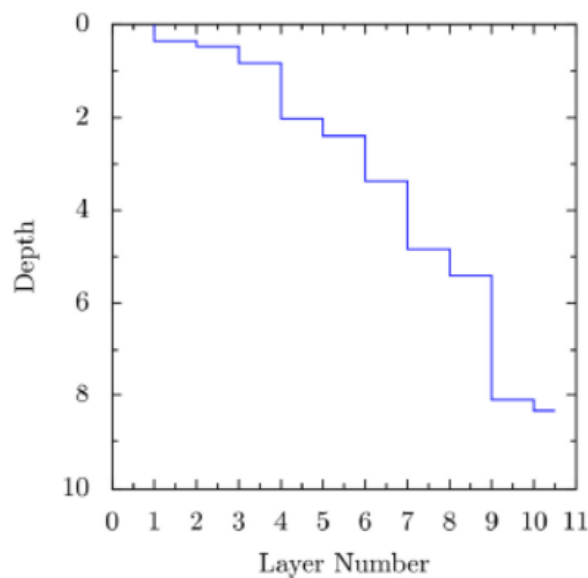
Figure 4.3.2 CDF of Poisson's distribution

+ A random layer thickness with an exponential distribution is generated by solving Eq. (2.3.2) with respect to thickness (h):

$$h = \frac{\ln[1-F(h;\lambda)]}{-\lambda} \text{ for } 0 < F(h;\lambda) \leq 1 \quad (2.3.3)$$

+ By randomly generating probabilities ($F(h;\lambda)$) with a uniform distribution between 0 and 1 and computing the associated thicknesses with Eq. (2.3.3), a layering profile was simulated for 10 layers with $\lambda = 1$ (Fig. 2.3.2).

An exponential distribution with $\lambda = 1$ will be referred to as a unit exponential distribution.

Figure 2.3.3 Ten-layer profile modeled by homogeneous Poisson process with $\lambda = 1$

+ Another way to think about generating exponential variables with a specific rate is to first generate a series of random variables with a unit exponential distribution and then convert them to a specific rate by dividing by the rate [see Eq. (2.3.3)].

This process is shown in Fig. 2.3.4; transforming from a constant rate of $\lambda = 1$ to a constant rate of $\lambda = 0.2$.

Fig. 2.3.4 and the associated layering are shown in Fig. 2.3.5.

In this example, the thicknesses (and depth) for $\lambda = 1$ (unit rate) are transformed to thicknesses (and depth) for $\lambda = 0.2$ (transformed rate).

Here, each thickness is increased by a factor of 5.0 ($1/\lambda$).

A similar technique is used to transform random variables generated with a unit exponential distribution into a nonhomogeneous Poisson process.

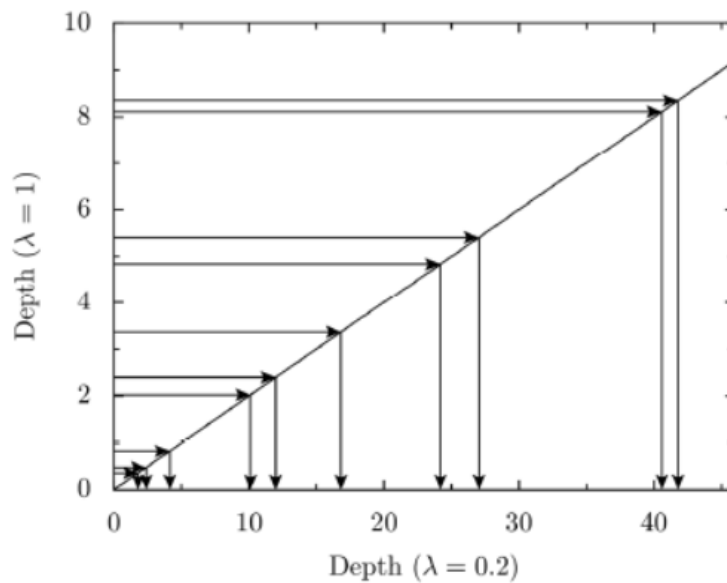


Figure 2.3.4 Transforming from constant rate of $\lambda = 1$ to constant rate of $\lambda = 0.2$

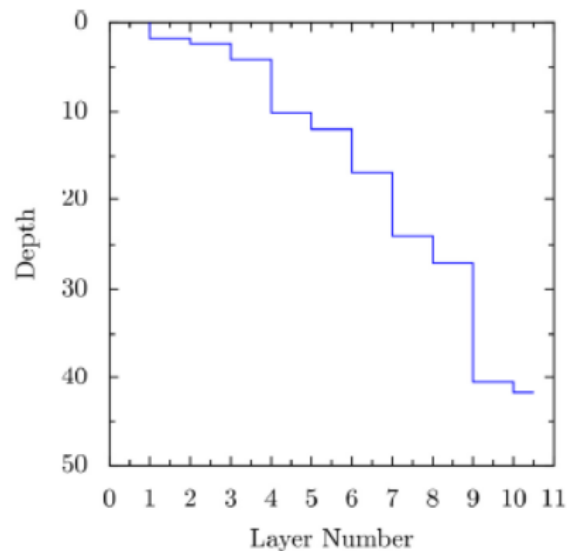


Figure 2.3.5 Ten-layer profile modeled by homogeneous Poisson process with $\lambda = 0.2$

2.3.2 Nonhomogeneous Layering

- + For a nonhomogeneous Poisson process with rate $\lambda(d)$, the cumulative rate $\Lambda(d)$ is defined as (Kao 1997):

$$\Lambda(d) = \int_0^d \lambda(s) ds \quad (2.3.4)$$

- + $\Lambda(d)$ represents the expected number of layers up to a depth d .

To understand the cumulative rate, consider a homogeneous Poisson process with a constant rate λ (i.e., $\lambda(s) = \lambda$).

In this case, Eq. (2.3.4) simplifies to $\Lambda(d) = \lambda d$.

For $\lambda = 1.0$ (unit rate), $\Lambda(d) = d$ such that the expected number of layers is simply equal to the depth.

For $\lambda = 0.2$ (transformed rate), $\Lambda(d) = 0.2d$, such that the expected number of layers is one-fifth the value of the unit rate because the layers are five times as thick.

This warping of the unit rate into a constant rate of 0.2 is represented by the straight line shown in Fig. 2.3.4.

- + Transforming between the y-axis and x-axis in Fig. 2.3.4 requires the inverse of the cumulative rate function.
- + For the homogeneous case,

$$\Lambda^{-1}(u) = u/\lambda \quad (2.3.5)$$

where u is the depth from an exponential distribution with $\lambda = 1.0$.

- + For the nonhomogeneous case, the inverse cumulative rate function is used to convert from a depth profile for $\lambda = 1.0$ (generated by a series of unit exponential random variables, u) to depth profile with a depth-dependent rate.

Before $\Lambda^{-1}(u)$ can be defined for the nonhomogeneous process, $\Lambda(d)$ and $\lambda(d)$ must be defined.

- + Toro (1995) proposed the following generic depth-dependent rate model:

$$\lambda_i(d_i) = a \cdot (d_i + b)^c \quad (2.3.6)$$

- + The coefficients a , b , and c were estimated by Toro (1995) using the method of maximum likelihood applied to the layering measured at 557 sites, mostly from California.

The resulting values of a , b , and c are 1.98, 10.86, and -0.89 , respectively.

The occurrence rate ($\lambda(d)$) quickly decreases as the depth increases (Fig. 2.3.6a).

This decrease in the occurrence rate increases the expected thickness of deeper layers.

The expected layer thickness (h) is equal to the inverse of the occurrence rate ($h = 1/\lambda(d)$) and is shown in Fig. 2.3.6b.

The expected thickness ranges from 4.2 m at the surface to 59 m at a depth of 200 m.

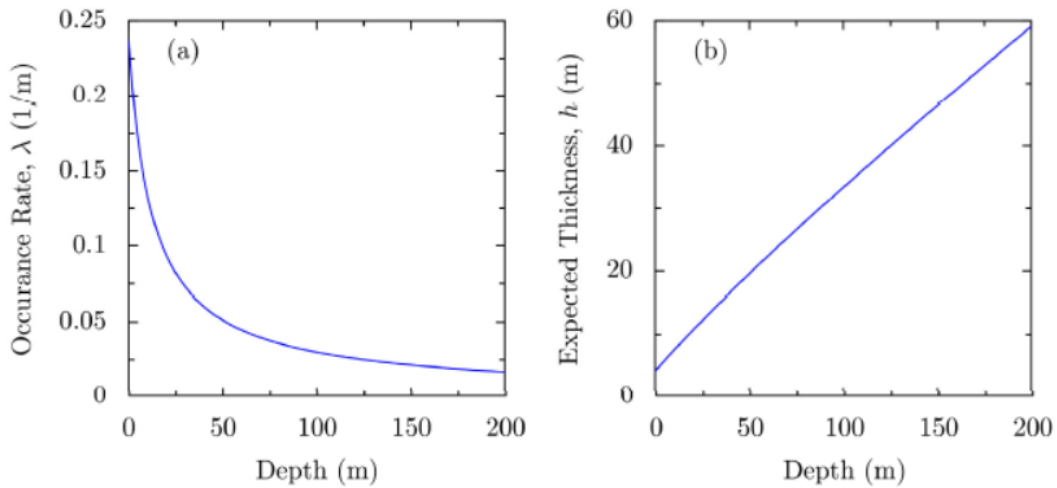


Figure 2.3.6 Toro (1995) layering model: (a) occurrence rate (λ) as function of depth (d), and (b) expected layer thickness (h) as function of depth (d)

+ Using Eqs. (2.5.4) and (2.5.6), the cumulative rate for the Toro (1995) modeled is defined as:

$$\Lambda(d) = \int_0^d a \cdot (s + b)^c ds = a \cdot \left[\frac{(d+b)^{c+1}}{c+1} - \frac{b^{c+1}}{c+1} \right] \quad (2.3.7)$$

+ The inverse cumulative rate function is then defined as:

$$d = \Lambda^{-1}(u) = \left(\frac{cu}{a} + \frac{u}{a} + b^{c+1} \right)^{\frac{1}{c+1}} - b \quad (2.3.8)$$

+ Using this equation a homogeneous Poisson process with $\lambda = 1.0$ (Fig. 2.3.3) can be warped into a nonhomogeneous Poisson process as shown in Fig. 2.3.7. The resulting depth profile is shown in Fig. 2.3.8.

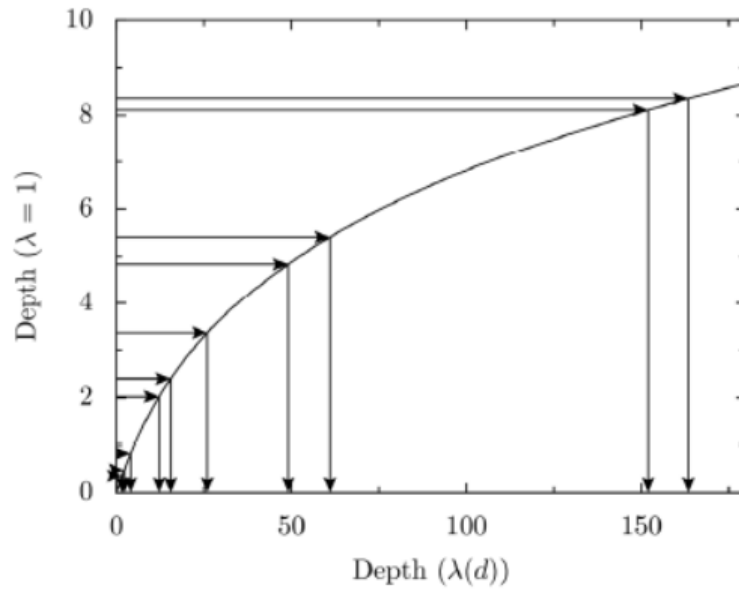


Figure 2.3.7 Transformation between homogeneous Poisson process with rate $\lambda = 1$ to the Toro (1995) nonhomogeneous Poisson process.

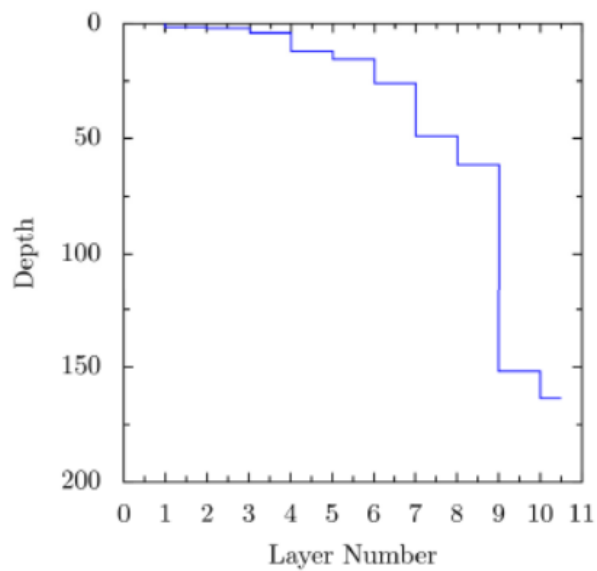


Figure 2.3.8 Layering simulated with nonhomogeneous Poisson process defined by Toro (1995).

2.3.3 Velocity

- Shear wave velocity in the i -th layer (V_i)

+ After the layering of the profile has been established, the shear wave velocity profile can be generated by assigning velocities to each layer.

In the Toro (1995) model, the shear wave velocity at mid-depth of the layer is described by a

lognormal distribution.

The standard normal variable (Z) of the i -th layer is calculated by:

$$Z_i = \frac{V_i - \ln[V_{median}(d_i)]}{\sigma_{\ln V_S}} \quad (2.3.9)$$

where V_i is the shear wave velocity in the i -th layer, $V_{median}(d_i)$ is the median shear wave velocity at mid-depth of the layer, and $\sigma_{\ln V_i}$ is the standard deviation of the natural logarithm of the shear wave velocity.

Eq. (2.3.9) is then solved for the shear wave velocity of the i -th layer (V_i):

$$V_i = \exp\{Z_i \cdot \sigma_{\ln V_S} + \ln[V_{median}(d_i)]\} \quad (2.3.10)$$

where

$Z_i = f(Z_{i-1}, \rho, \varepsilon_i)$: Standard normal random variable obtained from following equations

$\sigma_{\ln V_S} = f(\text{Soil Condition})$: Standard deviation obtained from following tables or field data

$V_{median}(d_i) = f(\text{Soil Condition})$: Median shear-wave velocity (m/sec) obtained from following tables or field data

+ Eq. (2.3.10) allows for the calculation of the velocity within a layer for a given median velocity at the mid-depth of the layer, standard deviation, and standard normal variable.

In the model proposed by Toro (1995), values for median velocity versus depth ($V_{median}(d_i)$) and standard deviation ($\sigma_{\ln V_S}$) are provided based on site class.

However, in the implementation of the Toro (1995) model in pShake, the median shear wave velocity is defined by the user.

Additionally, pShake includes the ability to truncate the velocity probability density function by specifying minimum and maximum values.

- Standard normal variable of the i -th layer (Z_i)

+ The standard normal variable of the i -th layer (Z_i) is correlated with the layer above it, and this interlayer correlation is also dependent on the site class.

The standard normal variable (Z_i) of the shear wave velocity in the top layer ($i = 1$) is independent of all other layers and is defined as:

$$Z_1 = \varepsilon_1 \quad (2.3.11)$$

where ε_1 is an independent normal random variable with zero mean and a unit standard deviation. The standard normal variables of the other layers in the profile are calculated by a recursive formula, defined as:

$$Z_i = \rho Z_{i-1} + \varepsilon_i \sqrt{1 - \rho^2} \quad (2.3.12)$$

where Z_{i-1} is the standard normal variable of the previous layer, ε_i is a new normal random variable with zero mean and unit standard deviation, and ρ is the interlayer correlation.

+ If $\rho = 1$, $Z_i = Z_{i-1}$.

If $\rho = 0$, $Z_i = \varepsilon_i$.

+ Correlation is a measure of the strength and direction of a relationship between two random variables.

The interlayer correlation between the shear wave velocities proposed by Toro (1995) is a function of both the depth of the layer (d) and the thickness of the layer (h):

$$\rho(d, h) = [1 - \rho_d(d)]\rho_h(h) + \rho_d(d) \quad (2.3.13)$$

where ρ_h is the thickness-dependent correlation and ρ_d is the depth-dependent correlation.

The thickness-dependent correlation is defined as:

$$\rho_h(h) = \rho_0 e^{(-h/\Delta)} \quad (2.3.14)$$

where ρ_0 is the initial correlation and Δ is a model fitting parameter.

As the thickness of the layer increases, the thickness-dependent correlation decreases.

The depth-dependent correlation (ρ_d) is defined as a function of depth (d):

$$\rho_d(d) = \begin{cases} \rho_{200} \left[\frac{d+d_0}{200+d_0} \right]^b & , d \leq 200 \\ \rho_{200} & , d > 200 \end{cases} \quad (2.3.15)$$

where ρ_{200} is the correlation coefficient at 200 m and d_0 is an initial depth parameter.

+ As the depth of the layer increases, the depth-dependent correlation increases.

The final layer in a site response model is assumed to be infinitely thick; therefore the correlation between the last soil layer and the infinite half-space is only dependent on ρ_d .

Toro (1995) evaluated each of the parameters in the correlation models (ρ_0 , ρ_{200} , A , d_0 , b) for different generic site classes presented in Table 2.3.3.

- + A site class is used to categorize a site based on the shear wave velocity profile and/or local geology. In the Toro (1995) model, the statistical properties of the soil profile (the median velocity, standard deviation, and layer correlation) are provided for two different classifications schemes, the GeoMatrix and $V_{s,30}$ classifications.

The GeoMatrix site classification classifies sites based on a general description of the geotechnical subsurface conditions, distinguishing generally between rock, shallow soil, deep soil, and soft soil (Table 2.3.1).

In contrast, the $V_{s,30}$ site classification is based on the time-weighted average shear wave velocity of the top 30 m ($V_{s,30}$) (Table 2.3.2), and requires site-specific measurements of shear wave velocity.

Table 2.3.1 Categories of geotechnical subsurface conditions (third letter) in GeoMatrix site classification (Toro, 1995)

Designation	Description
A	Rock Instrument is found on rock material ($V_s > 600$ m/s) or a very thin veneer (less than 5 m) of soil overlying rock material
B	Shallow (Stiff) Soil Instrument is founded in/on a soil profile up to 20 m thick overlying rock material, typically a narrow canyon, near a valley edge, or on a hillside
C	Deep Narrow Soil Instrument is found in/on a soil profile at least 20 m thick overlying rock material in a narrow canyon or valley no more than several kilometers wide
D	Deep Broad Soil Instrument is found in/on a soil profile at least 20 m thick overlying rock material in a broad canyon or valley
E	Soft Deep Soil Instrument is found in/on a deep soil profile that exhibits low average shear wave velocity ($V_s > 150$ m/)

Table 2.3.2 Site categories based on $V_{s,30}$ (Toro, 1995).

Average Shear wave Velocity
$V_{s,30}$ greater than 750 m/s
$V_{s,30} = 360$ to 750 m/s
$V_{s,30} = 180$ to 360 m/s
$V_{s,30}$ less than 180 m/s

- Standard deviation of the natural logarithm of the shear wave velocity ($\sigma_{\ln V_s}$) and Median shear wave velocity at mid-depth of the layer ($V_{median}(d_i)$)
- + Toro (1995) computed the statistical properties of the profiles for both the GeoMatrix and $V_{s,30}$ classifications using a maximum-likelihood procedure. The procedure used a total of 557 profiles, with 541 profiles for the $V_{s,30}$ USGS classification and only 164 profiles for the GeoMatrix classification. The standard deviation of the natural logarithm of the shear wave velocity ($\sigma_{\ln V_s}$) is presented in [Table 2.3.3](#) and the median shear wave velocities ($V_{median}(d_i)$) in are presented in [Table 2.3.4](#).

Table 2.3.3 Coefficients for Toro (1995) model

Property	GeoMatrix		$V_{S,30}$ (m/s)			
	A & B	C & D	>750	360 to 750	180 to 360	<180
$\sigma_{In V_S}$	0.46	0.38	0.36	0.27	0.31	0.37
ρ_0	0.96	0.99	0.95	0.97	0.99	0.00
ρ_{200}	0.96	1.00	0.42	1.00	0.98	0.50
Δ	13.1	8.0	3.4	3.8	3.9	5.0
d_0	0.0	0.0	0.0	0.0	0.0	0.0
b	0.095	0.160	0.063	0.293	0.344	0.744
Profiles	45	109	35	169	226	27

Table 2.3.4 Median shear wave velocity (m/s) based on generic site classification

Depth (m)	GeoMatrix		$V_{S,30}$ (m/s)			
	A & B	C & D	>750	360 to 750	180 to 360	<180
0	192	144	314	159	145	176
1	209	159	346	200	163	165
2	230	178	384	241	179	154
3	253	193	430	275	191	142
4	278	294	485	308	200	129
5	303	211	550	337	208	117
6	329	217	624	361	215	109
7.2	357	228	703	382	226	106
8.64	395	240	789	404	237	109
10.37	443	253	880	433	250	117
12.44	502	270	973	467	269	130
14.93	575	291	1070	501	291	148
17.92	657	319	1160	535	314	170
21.5	748	357	1260	567	336	192
25.8	825	402	1330	605	372	210
30.96	886	444	1380	654	391	229
37.15	942	474	1420	687	401	246
44.58	998	495	1460	711	408	266
53.2	1060	516	1500	732	413	289
64.2		541		749	433	318
77.04		566		772	459	353
92.44		593		802	486	392
110.93				847	513	435
133.12				900	550	
159.74					604	
191.69					676	
230.03					756	

- + Ten generated shear wave velocity profiles were created for a deep, stiff alluvium site using the two previously discussed methods.
- + In the first method, a generic site profile is generated by using the layering model coefficients and median shear wave velocity for a $V_{s,30} = 180$ to 360 m/s site class, shown in Fig. 2.3.9(a). This approach essentially models the site as a generic stiff soil site.
- + The second method uses the layer correlation for the $V_{s,30} = 180$ to 360 m/s site class, but the layering and the median shear wave velocity profile are defined from field measurements, shown in Fig. 2.3.8(b).

The site-specific layering tends to be much thicker than the generic layering as a result of the field measurements indicating thick layers with the same shear wave velocity.

In general both of the methods show an increase in the shear wave velocity with depth.

However, the site-specific shear wave velocity values are significantly larger than the generic shear wave velocity values.

At the surface, the generic site has a median shear wave velocity of 150 m/s compared to the site-specific shear wave velocity of 200 m/s.

At a depth of 90 m, the difference is even greater, with the generic site having a median shear wave velocity of 470 m/s compared to the site-specific median shear wave velocity of 690 m/s.

The difference in shear wave velocity is a result of the difference between the site-specific information and the generic shear wave velocity profile.

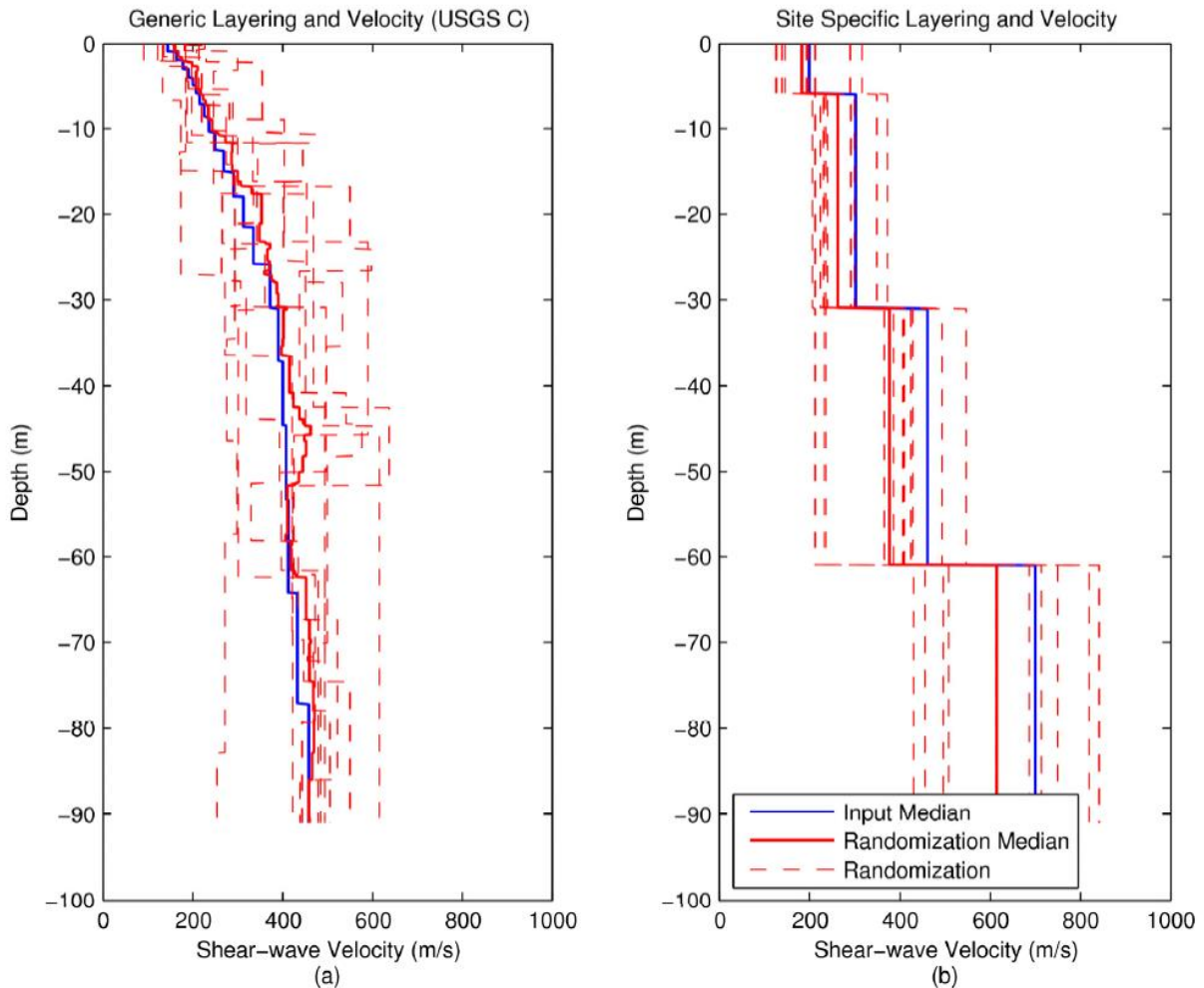


Fig. 2.3.9 Ten generated shear wave velocity (V_S) profiles for USGS C site class:
 (a) using generic layering and median V_S and (b) using user-defined layering and median V_S

2.4 Depth to Bedrock Model

+ The depth to bedrock can be modeled using either a uniform, normal, or lognormally distributed random variable.

When using the normal or lognormal distribution, the median depth is based on the soil profile.

The variation in the depth to bedrock is accommodated by varying the height of the soil layers.

If the depth to bedrock is increased, then the thickness of the deepest soil layer is increased.

Conversely, if the depth to bedrock is decreased then the thickness of this deepest soil layer is decreased.

If the depth to bedrock is less than the depth to the top of a soil layer, then the soil layer is removed from the profile.

2.5 Nonlinear Soil Properties Model (Daren, 2001)

+ The Darendeli (2001) empirical model assumes the variation of the properties follows a normal distribution.

The standard deviation of G/G_{max} and D varies with the magnitude of the property and is calculated with Eqs. (4.5.1) and (4.5.2), respectively.

$$\sigma_{NG} = 0.015 + 0.16\sqrt{0.25 - (G/G_{max} - 0.5)^2} \quad (2.5.1)$$

$$\sigma_D = 0.0067 + 0.78\sqrt{D(\%)} \quad (2.5.2)$$

+ Because the variation of the properties is modeled with a normal distribution that is continuous from $-\infty$ to ∞ , the generated values of G/G_{max} or D may fall below zero.

The most likely location for the negative values occurs when the mean value is small, which occurs at large strains for G/G_{max} and at low strains for D .

+ Negative values for either G/G_{max} or D are not physically possible; therefore the normal distributions need to be truncated.

To correct for this problem, minimum values for G/G_{max} and D are specified.

The default values in Strata are $G/G_{max} = 0.05$ and $D = 0.1\%$.

+ pShake also includes the ability to specify maximum values of G/G_{max} and D .

+ G/G_{max} and D curves are not independent.

Consider a soil that behaves more linearly, that is to say that the G/G_{max} is higher than the mean G/G_{max} .

During a loading cycle, the area inside the hysteresis loop would be smaller, which is indicative of less damping within the system.

Therefore, as the linearity of the system increases, the damping decreases.

To capture this effect, the soil properties are assumed to have a negative correlation with the default value set at -0.5 (i.e., $\rho = -0.5$).

+ To generate correlated G/G_{max} and D curves from baseline (mean) curves, the following expressions are used for each shear strain value in the curves:

$$\frac{G}{G_{max}(\gamma)} = \left[\frac{G}{G_{max}(\gamma)} \right]_{mean} + \varepsilon_1 \cdot \sigma_{NG} \quad (2.5.3)$$

$$D(\gamma) = [D(\gamma)]_{mean} + \rho \cdot \sigma_D \cdot \varepsilon_1 + \sigma_D \cdot \sqrt{1 - \rho^2} \cdot \varepsilon_2 \quad (2.5.4)$$

where ε_1 and ε_2 are uncorrelated random variables with zero mean and unit standard deviation, $[G/G_{max}(\gamma)]_{mean}$ and $[D(\gamma)]_{mean}$ are the baseline values evaluated at strain level γ , σ_{NG} and σ_D are the standard deviations computed from Eqs. (2.5.1) and (2.5.2) at the baseline values of $[G/G_{max}(\gamma)]_{mean}$ and $[D(\gamma)]_{mean}$, respectively, and ρ is the correlation coefficient between G/G_{max}

and D .

Eqs. (2.5.3) and (2.5.4) must be applied at different strain levels, but the same values of ε_1 and ε_2 are used at each strain level (i.e., perfect correlation between strain levels).

Using a correlation coefficient of -0.5, the nonlinear properties of sand ($PI=0$, $OCR=0$) at a confining pressure of 1 atm were generated 10 times, shown in Fig. 2.5.1.

Three of the realizations result in large shear modulus reduction curve relative to the mean.

Because of the negative correlation, the relatively high shear modulus reduction corresponds to a relatively low damping ratio.

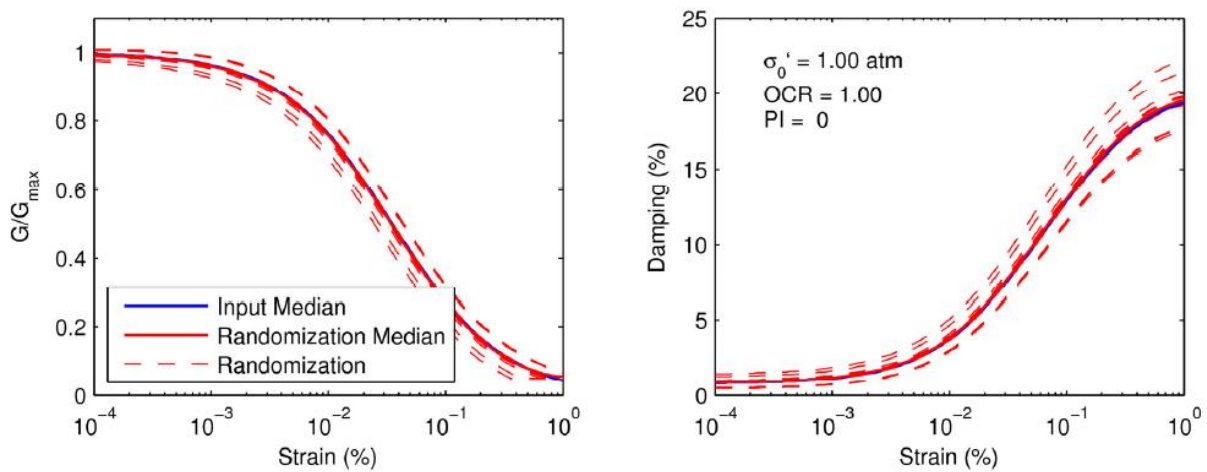
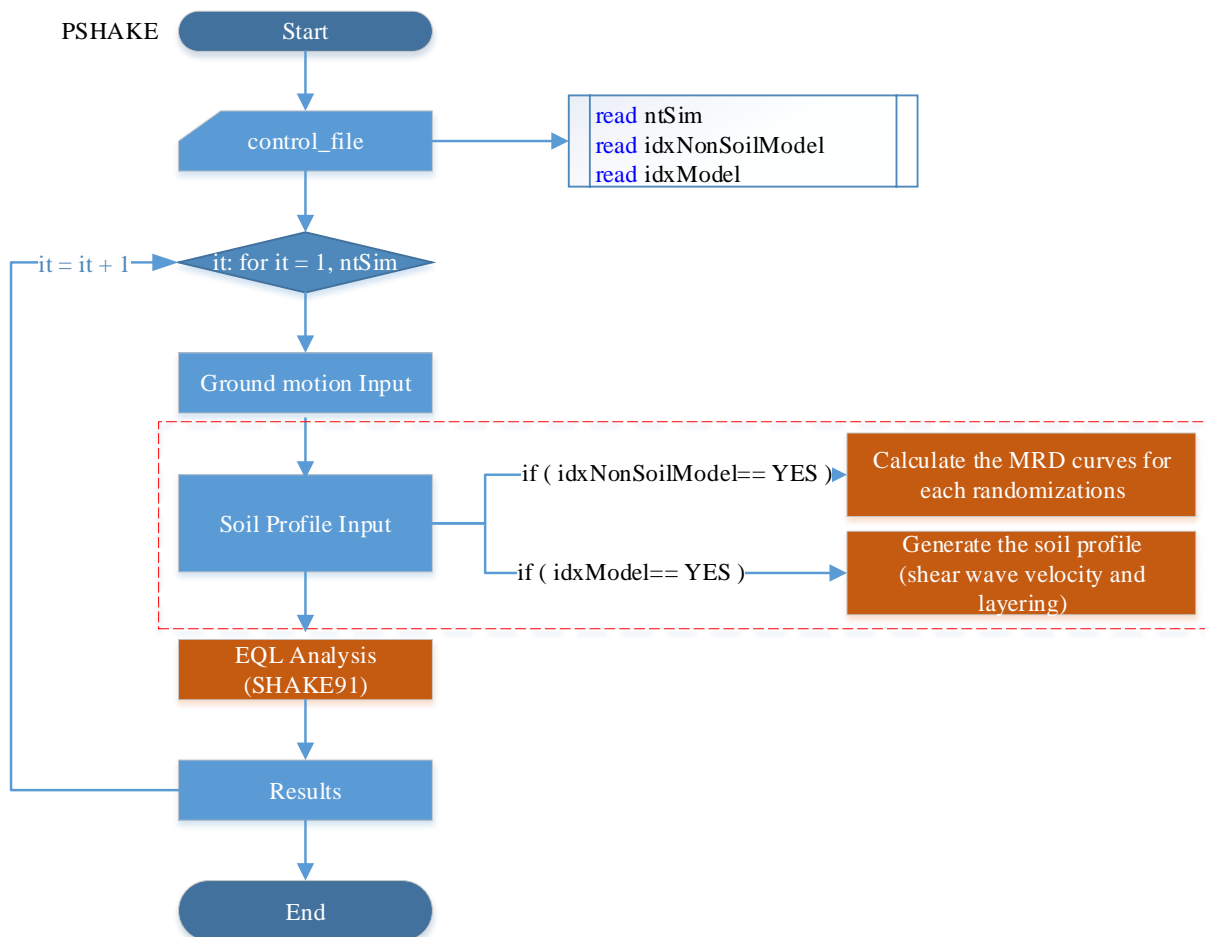
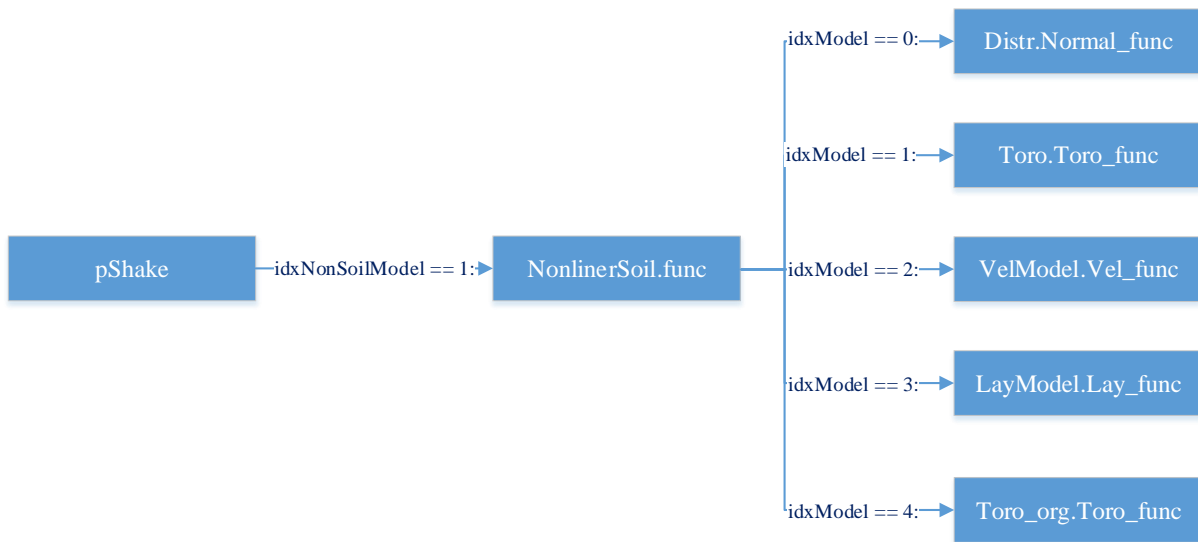


Fig. 2.5.1 Generated nonlinear properties assuming perfect negative correlation.

3. CODING

3.1 Flowchart





4. RUN

■ RUN

- [STEP 1] Input files

+ Refer to the following 'Section 3.1 Input'

- [STEP 2] Run pShake.exe

- [STEP 3] Output files

+ Refer to the following 'Section 3.2 Output'

Probabilistic Site Response Analyses (pSHAKE) with SHAKE91
Coded by Dookie Kim and Tran Thanh Tuan, <http://www.kim2kie.com>
Updated on December 1, 2017

5. MANUAL

Comments begin with "#(hash)"
[variable name] with explanations

- 1st line (Format: A15)

Column 1-15 [filename]
 Input filename of SHAKE91

- 2nd line (Format: I5)

Column 1-5 [ntSim]
 Number of total simulations

- 3rd line (Format: 2I5)

Column 1-5 [idxModel]
 None (Default) = 0
 Velocity Model (Option 2) + Layering Model (Option 3) = 1
 Velocity Model (Toro, 1995) (N or LN distribution) = 2
 Layering Model (N or LN distribution) = 3
 Velocity & Layering Model (Toro, 1995) = 4

Column 6-10 [idxSoilType]
 A & B = 1
 C & D = 2
 $750 \leq V_S(\text{m/sec})$ = 3
 $360 \leq V_S(\text{m/sec}) < 750$ = 4
 $180 \leq V_S(\text{m/sec}) < 360$ = 5
 $V_S(\text{m/sec}) < 180$ = 6

[Note] This is valid only for Velocity model (Toro, 1995)

i.e., idxModel = 1, 2, or 4

	[Note] Only available in case 'idxModel = 1 or 3'	
Column 11-15	[iCorVel] Value of ρ None (Default) from the table of Toro (1995)	= 0
	[Note] $V_i = \exp\{Z_i \cdot \sigma_{\ln V_S} + \ln[V_{median}(d_i)]\}$ $Z_i = \rho Z_{i-1} + \varepsilon_i \sqrt{1 - \rho^2}$	
Column 16-25	[coefToro(1)] None (Default)	= 1.98
Column 26-35	[coefToro(2)] None (Default)	= 10.86
Column 36-45	[coefToro(3)] None (Default)	= -0.89
	[Note] Toro (1995) proposed the following generic depth-dependent rate model : $\lambda_i(d_i) = a \cdot (d_i + b)^c$ The coefficients a , b , and c of a , b , and c are 1.98, 10.86, and -0.89, respectively that were estimated by Toro (1995) using the method of maximum likelihood applied to the layering measured at 557 sites, mostly from California.	
• 4th line (Format: 215)		
Column 1-5	[idxDist] Normal distribution (Default) Lognormal distributions	= 0 = 1
	[Note] This option is related to COV's of V and B.	
Column 6-10	[isigPlot] None (Default) Plot (+/- N or LN Standard Deviation)	= 0 = 1
• 5th line (Format: 215)		
Column 1-5	[idxNonSoilModel] None (Default) Daren Model (2001)	= 0 = 1
Column 6-10	[ntNonSoil] Number of nonlinear soils	

- 6th line (Format: 915): The following will be repeated for 'ntNonSoil' lines

Column 1-5	[idxNonSoil] Index of a nonlinear soil among the soil data in the SHAKE91 input
Column 6-10	[sPa] Atmospheric pressure in the same units as σ'_0 (p_a)
Column 11-15	[sSigop] Mean effective stress (σ'_0)
Column 16-20	[sOCR] Over-consolidation ratio (OCR)
Column 21-25	[sPI] Plasticity Index (PI)
Column 26-30	[sN] Number of cycles of loading (N)
Column 31-35	[sFreq] Frequency (f)
Column 36-40	[sRho] Correlation coefficient (ρ) between G/G_{max} and D
Column 41-45	[sDmax] Upper limit of Damping ratio (%) (D_{max})

- 7th line (Format: 315)

Column 1-5	[ntLayer] Number of total layers [Note] Same as nLayer of SHAKE91
Column 6-10	[idxTLayer] Type of Layers: Individuals (Default) = 0 Subgroups = 1
Column 11-15	[iSASSI] Files for SASSI input: None (Default) = 0 SIT and T14 files = 1
Column 16-30	[filenameSIT] Sample input file for SASSI input related to [iSASSI] Note: Blank means 'no sample file'

- 8th line and more lines (Format: I5, 3I10):

Column 1-5	[iLayer]
	idxTLayer = 0 in case of 'layer number' input
	idxTLayer = 1 in case of 'number of layers in each subgroup' input
Column 6-15	[COV(ntLayer,1)]
	Coefficient of variation (COV) of shear modulus (G)
	[Note] This doesn't work in case 'idxModel = 1'
Column 16-25	[COV(ntLayer,2)]
	Coefficient of variation (COV) of damping ratio (B)
Column 26-35	[COV(ntLayer,3)]
	Coefficient of variation (COV) of a layer thickness (H)

6. EXAMPLES

6.1 Input

- Input examples of 'pShake'

+ control.inp

```

shake.dat      # [A15] input filename of SHAKE91
5              # [ I5] ntSim
1 4 0         # [2I5] [Layering & Velocity] idxModel, idxSoilType, iCorVel
1 1           # [2I5] idxDist, isigPlot
1 1           # [2I5] [Nonlinear Soil] idxNonSoilModel, ntNonSoil
3 1.0 1.0 1.0 0.0 10.0 1.0 -0.5 15 # [9I5] idxNonSoil, ...
17 1 1        # [3I5] ntLayer, idxTLayer, iSASSI
2 0.1 0.2 0.2 # [4I5] iLayer, COV(ntLayer,1:3)~(G,B,H)
1 0.2 0.3 0.2
3 0.3 0.1 0.2
5 0.1 0.1 0.2
6 0.2 0.2 0.2

```

- Input examples of 'SHAKE 91'

+ shake.dat

```

option 1 -- dynamic soil properties - (max is thirteen):
Option 2 -- Soil Profile
Option 3 -- input motion:
Option 4 -- sublayer for input motion {within (1) or outcropping (0):
Option 5 -- number of iterations & ratio of avg strain to max strain
Option 6 -- sublayers for which accn time histories are computed & saved:
Option 6 -- sublayers for which accn time histories are computed & saved:
option 7 -- sublayer for which shear stress or strain are computed & saved:
option 7 -- sublayer for which shear stress or strain are computed & saved:
option 9 -- compute & save response spectrum:
option 10 -- compute & save amplification spectrum:
0

```

- Acceleration examples of 'SHAKE 91'

+ diam.acc

```

"Loma P. Eqk","Diamond Hts","H1_90","init. vel:"," .307 c/s","disp: -0.016 cm"
"Total No. of Points :",2000,"@ DT =",.02
"Peak Acceleration (g) =",.1128945,"@ Time (sec) :",10.92
-0.001694 -0.001668 -0.000086 -0.001356 -0.000678 0.000700 -0.001209 -0.000604
0.000730 0.000737 0.002496 0.004583 0.001644 0.001377 0.002408 -0.000352
:

```

6.2 Output

- Output files of 'SHAKE 91' for each running

+ shake_001.01

+ shake_002.01

+ :

```

1***** OPTION 1 *** READ RELATION BETWEEN SOIL PROPERTIES AND STRAIN
1***** OPTION 2 *** READ SOIL PROFILE
1***** OPTION 3 *** READ INPUT MOTION
1***** OPTION 4 *** READ WHERE OBJECT MOTION IS GIVEN
1***** OPTION 5 *** OBTAIN STRAIN COMPATIBLE SOIL PROPERTIES
1***** OPTION 6 *** COMPUTE MOTION IN NEW SUBLAYERS
1***** OPTION 6 *** COMPUTE MOTION IN NEW SUBLAYERS
1***** OPTION 7 *** COMPUTE STRESS/STRAIN HISTORY
1***** OPTION 9 *** COMPUTE RESPONSE SPECTRUM
1***** OPTION 10 *** COMPUTE AMPLIFICATION FUNCTION

```

+ shake_001.02

+ shake_002.02

+ :

```

1 17 0 Example -- 150-ft layer; input:Diam diam.acc
1 2 5.239 0.009 0.125 866.399
2 2 5.347 0.019 0.125 734.399
3 2 12.290 0.037 0.125 582.908
:
XMAX= 0.2044 diam.acc
ACCELERATION VALUES AT OUTCROPPING LAYER 1 - Example -- 150-ft layer;
input:Diam
-0.000008-0.000004-0.000014-0.000011-0.000030-0.000036-0.000097-0.000271 1
-0.001947-0.002533-0.001039-0.001557-0.001350 0.000282-0.000647-0.000490 2
0.001233 0.002253 0.004164 0.006675 0.004685 0.003038 0.003591 0.000889 3
:

```

+ shake_NonSoil_03_001.dat

Output file in case of Option of Nonlinear Soil Property [idxNonSoilModel = 1] (Ex: Line 5 in 'control.inp' file).

```

0.0001 0.9869 1.8255
0.0002 0.9835 1.8629
0.0003 0.9784 1.9231
0.0004 0.9710 2.0193
0.0007 0.9600 2.1718
0.0011 0.9438 2.4108
0.0018 0.9201 2.7799
0.0030 0.8859 3.3388
0.0048 0.8377 4.1641
0.0078 0.7725 5.3436
0.0127 0.6889 6.9577
0.0207 0.5889 9.0426

```


0.0336	0.4791	11.5473
0.0546	0.3696	14.3130
0.0886	0.2704	15.0000
0.1438	0.1883	15.0000
0.2336	0.1254	15.0000
0.3793	0.0801	15.0000
0.6158	0.0489	15.0000
1.0000	0.0282	15.0000

- Check files of 'pShake' for median values of responses

+ shake.chk

Median values Before SHAKE91

Depth(mid point,ft)	VS(fps)	DAMPING
2.011	1056.200	0.04090
4.687	977.400	0.04850
5.371	966.700	0.05010
5.402	966.700	0.05010
:		

Median values After SHAKE91

Depth(mid point,ft)	VS(fps)	DAMPING
2.011	1054.055	0.00700
4.687	974.158	0.00700
5.371	954.813	0.01100
5.402	954.813	0.01100
:		

Mean Amplification Spectrum

Frequency	Amplification
0.000	1.000
0.125	1.003
0.250	1.011
:	

Mean Response Spectrum

Period	DISP	VEL	ACC	PSU.ACC
0.010	0.00039	0.00989	0.15584	0.15583
0.020	0.00156	0.04365	0.15732	0.15729
0.030	0.00347	0.06621	0.15524	0.15524
0.040	0.00622	0.11258	0.15649	0.15644
0.050	0.00983	0.18350	0.15824	0.15821
:				

- Summary of the output in an Excel sheet

+ shake.xlsx

All the output data are saved in the form of the Excel spread sheet of which tabs are:

- Vs : Shear-wave velocity
- Damping : ...
- MAF : Mean Amplification Factor
- MRS : Mean Response Spectrum

- SITE input files of 'SASSI2000'

+ shake_001.SIT

+ shake_002.SIT

+ :

1	5.389	0.125	974.158	1822.483	0.007	0.007
2	4.827	0.125	846.382	1583.436	0.013	0.013
3	5.797	0.125	814.602	1523.981	0.019	0.019
⋮						
16	12.814	0.130	1300.543	2433.093	0.029	0.029
		0.140	2058.700	3851.475	0.012	0.012

- Earthquake time history input files of 'SASSI2000'

+ shake_001.T14

+ shake_001.T14

+ :

```

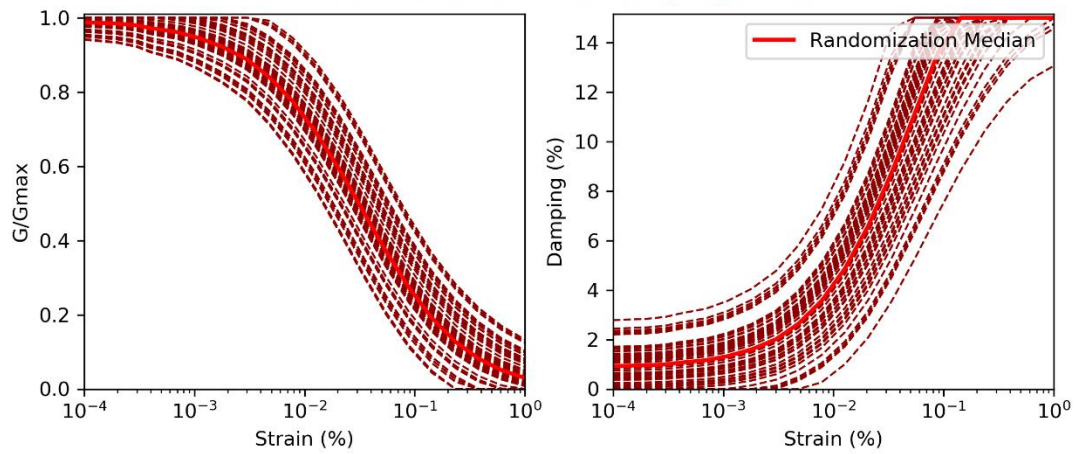
ACCELERATION VALUES AT OUTCROPPING LAYER 1- Example -- 150-ft layer;
input:Diam
0.000031-0.000053 0.000045-0.000114 0.000101-0.001115-0.003065-0.001247 1
-0.000325-0.001627 0.000299-0.000062-0.001438 0.000180 0.001904 0.002908 2
0.006120 0.006123 0.001596 0.002231 0.001847-0.001280-0.001899-0.000898 3
-0.000853-0.000633-0.003096-0.007483-0.006653-0.005470-0.002804 0.004377 4
0.008005 0.005687 0.000745-0.003360 0.000823 0.004221-0.003965-0.008746 5
-0.002400 0.004869 0.004013 0.001439 0.001271 0.002073-0.001993-0.005934 6
-0.002843-0.000060-0.000596-0.000550 0.001755 0.003168 0.000315-0.000291 7
0.001505 0.001082-0.001688-0.001416 0.001389 0.000265-0.000438 0.000368 8
-0.000481-0.003013-0.005510-0.004049-0.001410-0.002088-0.003668-0.000912 9
0.001706-0.000067-0.003626-0.004639 0.001431 0.005434 0.002352-0.001055 10
⋮
-0.000021-0.000060-0.000067-0.000001 0.000079 0.000166 0.000217 0.000268 225
0

```

- Graph files of 'pShake'

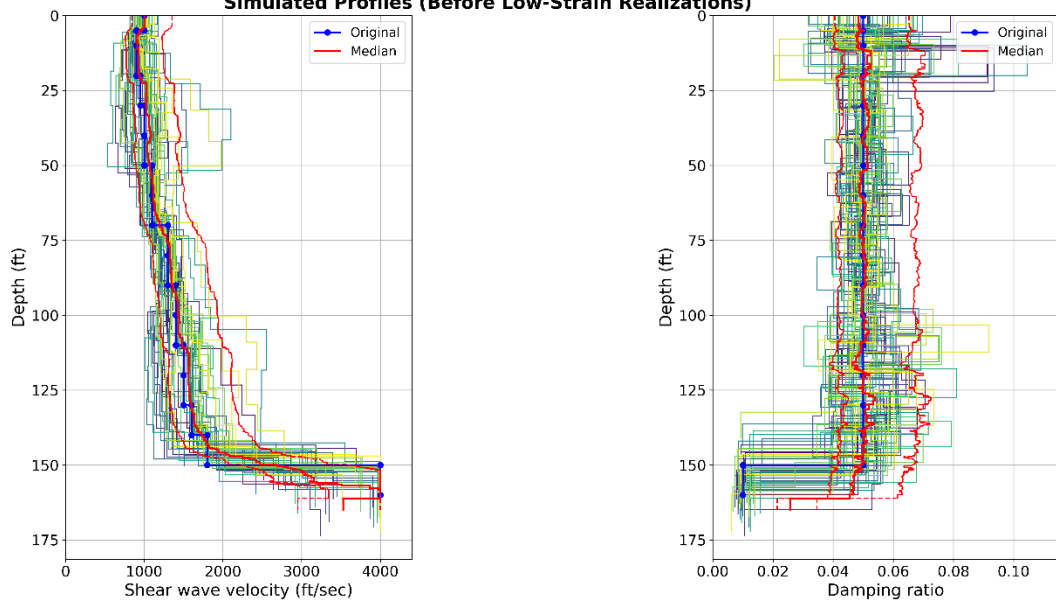
+ shake_Daren_soil_01.png

Generated nonlinear soil properties

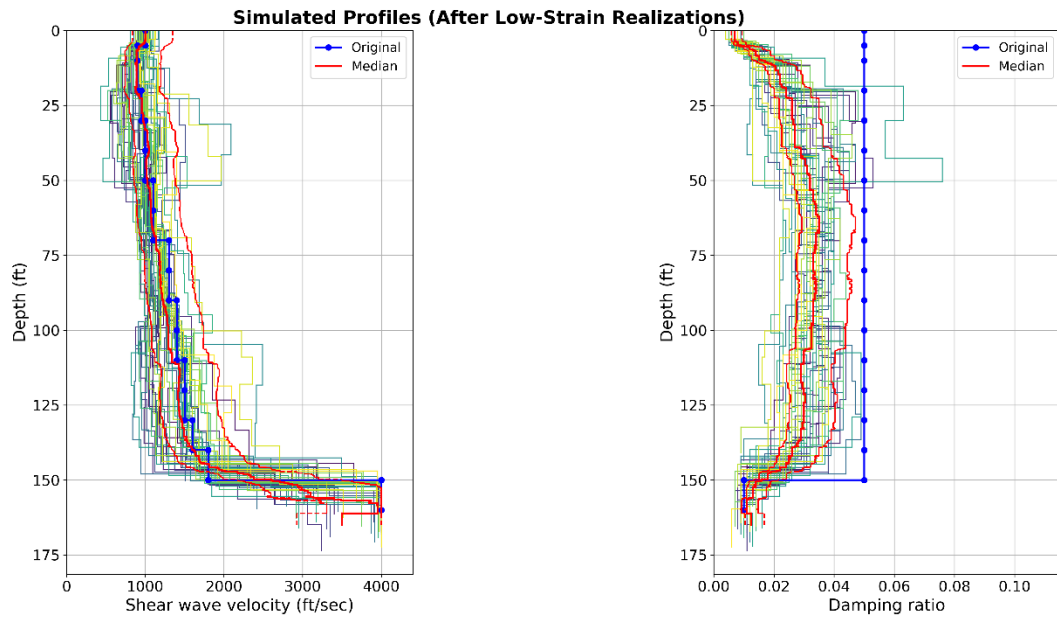


+ shake_fig1 (before realization).png

Simulated Profiles (Before Low-Strain Realizations)

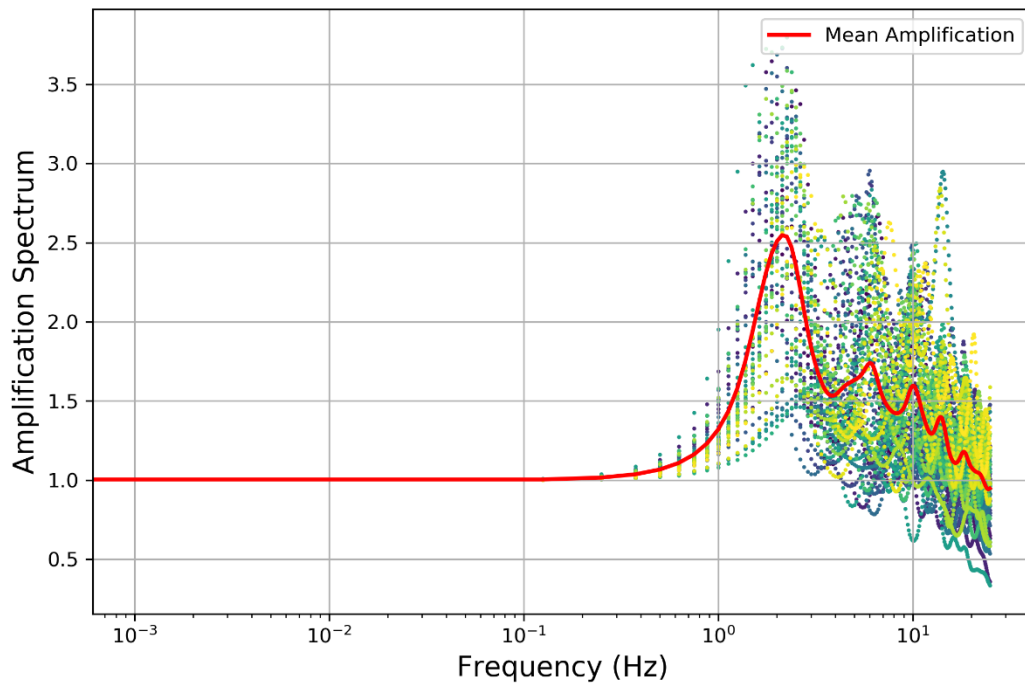


+ shake_fig2 (after realization).png



+ shake_fig3 (amplification spectrum).png

Amplification Spectrum



7. QUESTIONS AND ANSWERS

7.1 Question 1

기본적으로 SHAKE의 전단파속도 값은 Mean 값인지 아니면 Median 값인지요?
어느 경우에 Mean을 사용하고 어느 경우에 Median을 쓰는지 궁금합니다.

- Answer

자료의 분포를 보고, 대수분포의 경우, 중간값을, 정규분포의 경우 평균값을 써야 합니다.
자료의 개수가 충분하여 대수분포로 가정할 수 있다면 중간값을 사용하는 것이 합리적인 것입니다.
실제로 다수의 시층공별 자료를 사용하여, 각 층별 (기하)평균속도를 구합니다.
여기서 기하평균이란 대수분포의 산술평균과 동일하며, 일반 분포의 중간값에 해당합니다.

7.2 Question 2

PSHAKE Input 사항 중에 COV of G, B, H을 입력하도록 되어 있는데 해당 COV 값은
단순하게 '표준편차/평균값'을 나타내는 사항이 아닌가요?

- Answer

네. 맞습니다.

COV는 변동계수(Coefficient of Variation)을 나타냅니다.

이 값은 실험에 의해 구할 수도, 혹은 기준에 규정된 값을 사용할 수도 있습니다.

REFERENCES

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